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MODELLING THE DYNAMICS OF THERMOELASTIC STRUCTURES IN GRAPHS

ANNOTATION

for a dissertation for a degree Doctor of Philosophy (PhD)

according to the educational program

8D05403 "Mechanics"

Relevance of the research topic. The theory of thermoelasticity has been significantly developed in connection with the development and construction of various modern structures operating under conditions of intense dynamic and thermal loads. Thermoelasticity, being a generalization of the classical theory of elasticity and the theory of thermal conductivity, describes a wide range of applied problems. In construction and mechanical engineering, rod structures are widely used as supports for buildings and structures, as connecting and transmission links of various mechanisms and machines. During operation, they are exposed to various thermal and mechanical effects that create a complex stress-strain state in structures and constructions. Under these conditions, the requirements for reliability and safety also increase. Determining the thermal stress state of rod structures (RS) is an urgent scientific and technical task.

The aim of the work. The study of the stress-strain state of thermoelastic rod structures in graphs taking into account their physical, mechanical and geometric parameters under the influence of non-stationary and periodic force and thermal loads.

Research objectives:

- Construction of fundamental and generalized solutions of the equations of thermoelastic rod dynamics. Construction of the Green's tensor of the equations of uncoupled thermoelasticity, numerical implementation and study of its properties.
- Construction and solution of the Cauchy problem of the dynamics of an infinite thermoelastic rod by V.S. Vladimirov's method. Numerical implementation, calculation of the thermal stress-strain state of the rod and analysis of the solution under the action of distributed force and heat sources.
- Formulation and analytical solution of eight boundary value problems of the dynamics of finite-length thermoelastic rods by the method of generalized functions under periodic and non-stationary external force and heat effects.
- Numerical implementation of the first and second boundary value problems of the dynamics of a thermoelastic rod under periodic external force and heat effects. Calculation of displacements and temperature for various materials under periodic fluctuations in temperature and stress at the ends of the rod. Determination of the thermal stress state and its analysis.
- Statement and analytical solution of temperature boundary value problems on an N-star thermal graph with Dirichlet and Neumann boundary conditions at its ends.
- Statement and analytical solution of boundary value problems on an N-star elastic graph with Dirichlet and Neumann boundary conditions at its ends.
- Statement and analytical solution of boundary value problems of the dynamics of a thermoelastic N-star graph with Dirichlet and Neumann boundary conditions at its ends.
- Numerical implementation of the solution of boundary value problems of the dynamics of a thermoelastic two-link graph with given periodic displacements of the graph ends and temperatures at its ends. Determination of the thermostressed state and analysis of thermoelastic parameters of graph links and different frequencies of temperature oscillations, heat fluxes, displacements and stresses at its ends.

- A solution to the Cauchy problem of thermoelastic rod dynamics is constructed. Numerical implementation is performed, the thermal stress-strain state of the rod is calculated and the solution is analyzed.

- Eight boundary value problems of the dynamics of a finite-length thermoelastic rod are posed under periodic and non-stationary external force and thermal effects at its ends. Their solutions are constructed in analytical form using the method of generalized functions.

- Numerical implementation of the first and second boundary value problems of the dynamics of a thermoelastic rod under periodic external force and thermal effects is carried out. Displacements and temperature of the rod are calculated under periodic fluctuations of temperature and stress at the ends of the rod. Its thermal stress state is determined. The calculations performed in dimensionless parameters allow us to determine the thermal stress state of a rod made of various materials, taking into account their thermodynamic parameters.

- The first boundary value problem of thermodynamics of an N-star graph is stated for given non-stationary and periodic displacements of the graph ends and temperatures at its ends. Its analytical solution is constructed in the space of Fourier transforms over time, which determines the temperature and displacement on each edge of the graph.

- A numerical implementation of the solution to the first boundary value problem of the dynamics of a thermoelastic two-link graph is performed for periodic oscillations of displacements and temperatures at its ends. A multiparameter analysis of the solution is performed for different oscillation frequencies and thermoelastic parameters of the medium.

- The second boundary value problem of thermodynamics of an N-star thermoelastic graph is stated for given non-stationary and periodic stresses and heat fluxes at its ends. Its analytical solution is constructed in the space of Fourier transforms over time, which determines the temperature and displacement on each edge of the graph.

- A numerical implementation of the solution of the second boundary value problem of the dynamics of a thermoelastic two-link graph with periodic oscillations of stresses and heat flows at its ends was performed. A multiparameter analysis of the solution was carried out for different oscillation frequencies and thermoelastic parameters of the medium.

The reliability and validity of scientific positions, conclusions and results of the dissertation. The reliability of the results is confirmed by the use of classical mathematical models of thermoelasticity, strict analytical methods for solving the posed boundary value problems of thermodynamics of rod structures with the coincidence of theoretically constructed solutions with the results of their computer implementation in the MatCad 15 system with high accuracy at performing numerical experiments for different geometric and physical-mechanical parameters of structures.

Theoretical and practical significance. Rod elements are widely used in various fields of technology. Rod systems (trusses) are widely used in structural mechanics as supporting structures for bridges and power lines. Their main advantage is high load-bearing capacity with low structural weight. Rod structures are widely used in mechanical engineering as connecting and transmission links of various mechanisms and machines. During operation, these systems are subject to various thermal and mechanical effects, which create a complex stress-strain state in these structures. To determine the thermal stress state of rod structures in a wide range of external force and thermal effects, it is necessary to develop effective mathematical research methods based on mathematical modeling of their thermodynamics using various models of thermoelastic media. The use of mathematical modeling methods makes it possible to conduct research into physical processes occurring in rod structures and their elements, taking into account their thermophysical characteristics, and to determine their thermodynamic state at the design stage, which, in turn, is the basis for predicting the behavior of the product under specified operating conditions. In this case, it is important not only to develop calculation models, but also to develop effective algorithms for studying models.

The development of analytical and numerical methods aimed at mathematical modeling of rod structures and network structures is based on the analysis of the corresponding direct and inverse problems for systems of equations with distributed parameters on graphs. First of all, this is the

Subject of the research. The subject of the research is mathematical models of rod structures based on mathematical models of uncoupled thermoelasticity, which are described by systems of hyperbola-parabolic partial differential equations and boundary value problems for them.

The object of the study is thermoelastic rods and thermoelastic rod structures – thermoelastic graphs.

Research methods. Mathematical models of elasticity and thermoelasticity theory, theory of partial differential equations of hyperbolic, parabolic and parabolic-hyperbolic types, theory of generalized functions, Fourier and Laplace transforms, methods of linear algebra, method of generalized functions, V. S. Vladimirov's method, computer experiments in the MatCad -15 system.

Scientific novelty of the work.

Using the theory of generalized functions, the Green tensor for the equations of uncoupled thermoelasticity in the spatially one-dimensional case is constructed and its properties are investigated. Generalized solutions of the equations of the dynamics of a thermoelastic rod under arbitrary thermal and force effects from the class of generalized functions of slow growth are constructed, which allows us to study the thermally stressed state of rods under the effects of impact force and thermal loads.

A method of generalized functions has been developed for solving non-stationary and stationary boundary value problems of thermodynamics of rods of finite length, and analytical solutions have been constructed for eight boundary value problems under periodic and non-stationary external force and thermal effects.

The numerical implementation of the first and second boundary value problems of the dynamics of a thermoelastic rod under periodic external force and thermal effects is performed. The displacements and temperature field of the rod are calculated, its thermal stress state is determined. The use of dimensionless parameters allows generalizing the results for materials with different thermodynamic characteristics.

Based on the generalized function method (GFM), a technique for solving boundary value problems on an N-star heat graph at given temperatures or heat flows at its ends has been developed. Resolving systems of equations have been constructed to determine the temperature of each graph link for any geometric and thermal parameters of the graph links. Analytical calculation formulas have been constructed that allow one to study the thermal state of the graph with periodic oscillations. For non-stationary problems, the inverse Fourier transform is used.

Based on the GFM, a method for solving boundary value problems on an N-star elastic graph with given displacements or stresses at its ends has been developed. Resolving systems of equations have been constructed to determine displacements, deformations, and stresses of each graph link for any geometric and elastic parameters of the graph links. Analytical calculation formulas have been constructed that allow one to study the stress-strain state of an elastic graph with periodic oscillations. For non-stationary problems, the inverse Fourier transform is used.

Using solutions of boundary value problems on thermal and elastic graphs, a method has been developed for solving stationary and non-stationary boundary value problems on an N-star thermoelastic graph for given displacements or stresses, temperatures or heat flows at its ends.

To test the developed method, a numerical implementation of solutions to boundary value problems with Dirichlet and Neumann conditions on thermoelastic rods of finite length and two-link thermal and elastic graphs was carried out. The numerical experiments showed high accuracy of the developed method for solving problems on graphs. The advantage of this method is the possibility of using it on graphs of any structure. For example, linear and network structures.

Scientific provisions submitted for defense.

- Fundamental and generalized solutions of the equations of thermoelastic rod dynamics are constructed. Green's tensor of the equations of uncoupled thermoelasticity is constructed and its properties are studied. Numerical implementation of the tensor is carried out and its properties are studied for different thermoelastic parameters.

analysis of spectral completeness and the basis property of eigenfunctions of the corresponding boundary value problems in the space of square integrable functions, as well as finding conditions for the uniqueness of solutions of inverse problems acceptable for practical implementation. The history of boundary value problems for differential equations on graphs is relatively short. Most of the works are devoted to problems of spectral theory (G. Lumer, Yu. Belov, S. Nikaiz, O. Penkin, G. Borg, B. Levitan, etc.). The study of the wave equation on graphs was carried out by F. Ali-Mehmeti, C. Cattaneo and L. Fontana, J. Friedman and J-P. Tilly and others. In recent decades, most of the works have been devoted to controllability, observability and stabilization of elastic systems (see the works of Cox and S. Suazua and other authors). Mathematical modeling of the process of heat propagation in a system of rods on a tree-type graph in the form of a bundle of linear differential operators was performed by Yu. Martynov. At present, the theory of boundary value problems on graphs of various structures is beginning to develop intensively.

It should also be noted that studies of the thermal stress state of rods of finite length with different physical and mechanical properties are conducted by only a few authors, and mainly on the basis of numerical finite element and finite difference methods for studying the static and quasi-static stress state of rods in the works of Kudaykulova A., Zhumadillaev A., Tasheva A., Mishenko A. and others.

In the formulation of the thermodynamics problem on star graphs presented in the dissertation, no one has previously posed or solved it. These studies are new and serve as a theoretical contribution to the development of the theory of thermoelasticity, thermodynamics of rods and rod structures, as well as the development of methods of mathematical physics for solving boundary value problems for hyperbolic-parabolic systems on differential graphs.

The proposed here method of modeling and calculating the thermal stress state of structural elements of composite rod structures, based on the use of the method of generalized functions on rods of finite length, made it possible to obtain analytical formulas that are very convenient for use in engineering practice in the design and study of the strength properties of such structures. Note that this model of thermoelasticity is widely used in the standards for the design of building structures, machines and mechanisms, therefore the research conducted in the dissertation will be in demand in practice.

Structure and volume of the dissertation. The volume of the dissertation of 110 pages contains: introduction, three chapters, conclusion and bibliography of 72 titles.

Main content of the dissertation. The dissertation develops mathematical methods for modeling the thermodynamics of rod structures on non-uniform star graphs.

The first section of the dissertation considers a model of a thermoelastic rod using the equations *of uncoupled thermoelasticity.* The problem is set of constructing fundamental solutions to the equations of uncoupled thermoelasticity in a spatially one-dimensional case describing the dynamics of thermoelastic rod structures under the action of force and heat sources of disturbances. Based on the theory of the Fourier transform of generalized functions, the Green's tensor and generalized solutions of the system of thermoelasticity equations for arbitrary mass forces and heat sources are constructed. A computer implementation of the Green's tensor and a number of constructed solutions is carried out in the MatCad 15 system. Graphs of numerical experiments describing displacements, temperature and stresses in the rod under various external influences are given. The Cauchy problem for the system of equations of uncoupled thermoelasticity is considered. Using the method of V.S. Vladimirov, the problem is set in the space of generalized functions and its generalized solution is obtained. Its regular integral representation is constructed.

In the second chapter, based on the GFM, a method is developed for solving boundary value problems for rods of finite length under various boundary conditions for displacements, stresses, temperatures and heat flows both at its ends and along the length of the rod, which can be used to study various rod structures under thermal heating conditions. Advantages of the GFM allows you to obtain a resolving system of equations that includes the initial and boundary conditions of all the problems set. Therefore, there is no need to solve each boundary value problem separately. The peculiarity of the constructed solutions makes them convenient for engineering calculations, since it allows you to study the effect of each boundary condition at the ends of the rod on its thermal stress state. The peculiarity of the constructed solutions makes them convenient for solving boundary value problems on multi-link graphs.

In the third chapter, boundary value problems on a thermoelastic star graph are considered, which can be used to study various rod structures under thermal heating-cooling conditions. Based on the GFM, solutions of non-stationary direct boundary value problems of thermoelasticity are constructed under the action of force and heat sources of various types on a star graph. Resolving systems of equations and regular integral representations of generalized solutions are obtained, which give an analytical solution to the posed boundary value problems.

The relationship of this work with other research works. The work was carried out within the framework of the projects of the grant funding program for fundamental research in the field of natural sciences: "Boundary value problems of the dynamics of deformable solid and electromagnetic media and their solution" (2018-2020, AP05132272, № GR 0118PK00479, "Study of initial-boundary value problems of the wave equation on graphs" (2021-2023, AP09261033, № GR 022PK00530), "Modeling of thermal and wave processes in thermoelastic rod structures on graphs" (2024-2026, AP23488145, № GR 0124PK00588).

Publications. 10 articles have been published on the topic of the dissertation. 3 of them are in scientific journals indexed by the Scopus database, and 3 articles are in domestic journals recommended by COXON and Science of the Ministry of Education and Science of the Republic of Kazakhstan. The main results of the work have been presented at 13 international conferences and congresses..

Personal contribution of the author. The formulation of boundary value problems and research methods were proposed by the scientific and supervisors. Construction of solutions to boundary value problems, development of algorithms and software implementation of their solutions were carried out by the author of the dissertation under their supervision. Analysis of the obtained solutions and preparation of articles based on the research materials were carried out jointly with the scientific and supervisors. Design of articles and presentation of their publications were carried out by the author. The results of research on the topic of the dissertation at international conferences were also presented by the author.

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